MATH 6 – Summary of Math 6 standards (2016)

Teachers: Please let me know if this guide is helpful, or how it could be improved.

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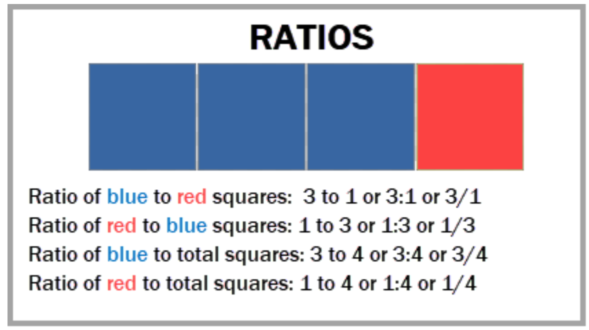
6.1 RATIOS



A **ratio** is a comparison between quantities
A ratio can be written as:

- a **fraction** – 2/3
- using a **colon** – 2:3
- using the word "**to**" – 2 to 3

In a classroom with 5 students with 2 girls and 3 boys

- The ratio of **girls** to **boys** is 2:3 or 2/3 or 2 to 3.
- The ratio of **boys** to **girls** is 3:2, or 3/2 or 3 to 2. Note that **ORDER MATTERS**.
- The ratio of **girls** to the **total** number of students is 2:5 or 2/5 or 2 to 5.
- The ratio of **boys** to **total** students is 3:5 or 3/5 or 3 to 5.

Note that in this example a ratio can be used to compare **2 parts** or a **part to a whole**.

?? How many **girls** in a classroom of **10 students** (same ratio of girls to boys - 2:3)? Answer – 4. Why? 2/3 = 4/6

?? How many **boys** in a classroom of **25 students**? Answer- 15. Why? 2/3 = 10/15

If the ratio of **boys** to the **total** number of students is **1:3,**

?? What is the ratio of **boys** to **girls**? Answer 1:2 (total = 3, 1 boy, 2 girls)

?? If there are 3 **boys**, how many **total** students are there? Answer – 9 (Why? 1/3 = 3/9)

?? If there are 5 **boys**, how many **girls** are there? Answer – 10 (Why? ½ = 5/10)

6.2 Compare fractions, mixed numbers, decimals, and percent.

Percent (%) means "**per 100**". 10% = 10/100 = 1/10 = .1

?? What is 3/5 written as a percent and decimal? Answer: 3/5 = 6/10 = 60/100 = 60% = .60 = . 6

?? What is 73% written as a fraction and a decimal? Answer – 73/100 = .73

?? What about 162%? Answer - 162/100 = 1.62 (note that **100% = 1**, so more than 100% is more than 1)

When comparing fractions, it's often helpful to note if they are greater or less than ½ (example 3/7 < 4/8 < 5/9), or how close they are to 1 (6/7 < 7/8 < 8/9 < 9/10 because 1/7 > 1/8 > 1/9 > 1/10).

¼ = 25/100 = .25

Repeating decimals: 2/9 = 20/90 = 21/99, but as a decimal, the final 2 continues to repeat. This is written as .222**...** or **0.$\overline{2}$** to indicate a repeating 2. The **ellipse (...)** or **overline** means **repeating**.

6.3 INTEGERS

Integers are **whole numbers** and can be positive or negative.

When comparing two **negative** integers, the one **closer to zero is greater** (-3 > -10 > -22).

Absolute value of an integer, written with **symbol | |** is its distance from zero on the number line. |-6| = 6, and |6| = 6. Absolute values are **always positive**. The absolute value of zero is zero.

6.4 EXPONENTS

Any number (except 0) raised to **zero power is 1**. A positive number raised to the **one power** is the **number itself.**

5^3 - 5 is the **base,** 3 is the **exponent** = **5 x 5 x 5**

To indicate **multiplication,** a dot • can be used in place of **x**. 3^4 = 3 • 3 • 3 • 3 = 81

A **perfect square** is a number with a square root that is a whole number **–** These numbers are **perfect squares - 4, 9, 16, 25, 36, 49, 64, 81, 100.**

6.5 COMPUTATION – FRACTIONS, MIXED NUMBERS AND DECIMALS

FRACTIONS

Fraction in **simplest form –** divide numerator and denominator by **greatest common factor.** 10/15 = 2/3 (divide num. and den. by 5)

Multiply 2 fractions – multiply numerators to get numerator**, denominators** to get denominator**. ¼ x ½ = 1/8**

Multiply a fraction and a whole number - ½ x 8 = ½ x 8/1 = 8/2 = 4/1 = 4

Divide a fraction by another is the opposite, so **flip** the second fraction **–** ½ ÷ ¼ = ½ x 4/1 = 4/2 = 2/1 = 2

Check or **estimate** your answer – when you multiply 2 fractions, you end up with a **part of a part** – so the answer will be **less**. For example, ½ x ½ = ¼.

DECIMALS

Estimate first, then **calculate**. Estimation will help you know if your answer is reasonable. Try these (answers below):

a. **10 x .1**
b. **10 ÷ .1**

Answers -

a. **1** Why? 10/1 x 1/10 = 10/10 = 1/1 = 1
b. **100** Why? 10/1 ÷ .1 = 10/1 ÷ 1/10 = 10/1 x 10/1 = 100/1 = 100

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c. **Estimate then calculate 1.12 x 4.21**
d. **Estimate then calculate 1.12 ÷ 4.21**

Answers -

c. estimate - over 4, answer 4.71 Why? 1 x 4 = 4 so answer will be greater than 4.
d. estimate - about ¼ answer .26 Why? 1 ÷ 4 = 1 x ¼ = approx. ¼

6.6 COMPUTING WITH INTEGERS

ORDER OF OPERATIONS

P P - parenthesis ()
E E - exponents 4^2
M
D MD - multiplication, division (left to right)
A
S AS - addition, subtraction (left to right)

Try this:

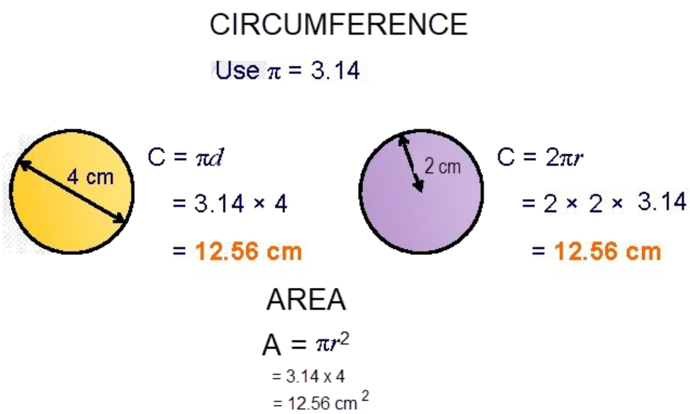
|-3|(6 - 2^2) 4 - 2 =

Answer – 22 – if you came up with a different answer, look at the order of operations below

|-3|(6 - 22) 4 - 2 = 3(6 - 2^2) 4 - 2 = 3(6 - 4) 4 - 2 = 3 (2) 4 – 2 = 6 x 4 - 2 = 24 - 2 = 22

PROPERTIES - This standard mentions a number of **properties** that students should be familiar with, but do NOT necessarily have to name. It's more important to understand what they do and how they help with equations. These properties are listed again with a few others in standard 6.13, so I have not listed them here. Scroll to the last page for a list of all the properties students should be familiar with.

6.7 CIRCUMFERENCE AND AREA OF CIRCLES, PERIMETER OF TRANGLES AND RECTANGLES



CIRCLES

The value of **pi (π)** is the **ratio** of the **circumference** of a circle to its **diameter**.

pi (π) = approx. **3.14** or 22/7.

Circumference of a circle: **C = πd** (d is diameter) or **C = 2πr** (r is radius) . **Circumference** is approx. **three times** the **diameter**.

Area of a circle: **A = πr²** (r is radius)

?? What is the approx.. circumference and area of a circle with the radius of 3 cm ?

Answer - Approx **Circumference** = over 18 (6 x 3) More exact: 6(diameter) x 3.14(pi) = 18.83 cm.

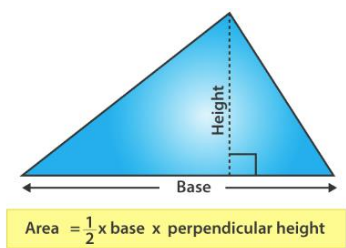
Area: approx. - r^2 = 9 so πr^2 is over 27 . More exact 9 x 3.14 = 28.26 cm^2

SQUARE – length of each **side** is **5cm**

- **Perimeter**: ***4s*** where *s* is side = 20cm
- **Area**: **S^2** = 25cm^2

RECTANGLE – width = 3 cm, length = 4 cm

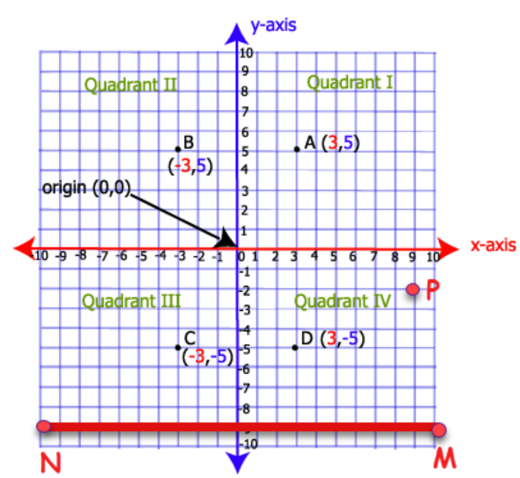
- Perimeter: **2w + 2l** = 14 cm
- Area: **length ∙ width** = 12 cm^2



TRIANGLE

- **Perimeter**: add length of 3 sides
- **Area**: **A = ½ base x height**

6.8 COORDINATE PLANES



Points represented as **(x, y)**

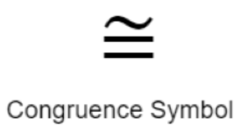
What are the coordinates of P? answer: (9, -2)

Know the **origin (0, 0), x-axis**, **y-axis** and **quadrants I – IV** (counterclockwise)

What do all of the points an line NM share in common? For every point, y = -9.

Points with the same **x coordinate** form a **vertical line**.

6.9 CONGRUENCE OF SEGMENTS, ANGLES, AND POLYGONS



The symbol for **congruency** is ≅

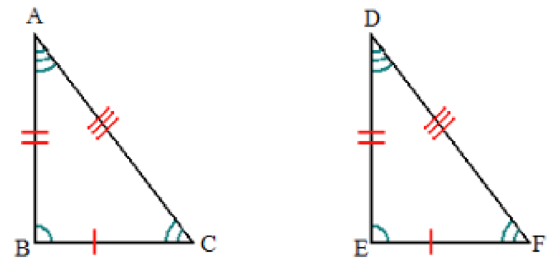
Congruent figures have **exactly the same size and shape**.

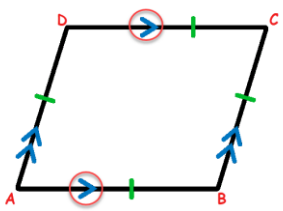
Congruent line segments have the same **length**.

Congruent angles have the same **measure** (in degrees)

Congruent polygons have the same number of sides, and the **corresponding sides and angles are congruent**.

These triangles are congruent. The hatch and angle marks indicate that line BC ≅ line EF, line BA ≅ line ED, angle D ≅ angle A etc.





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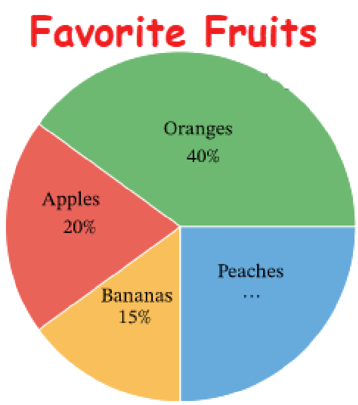
The **arrows** show that line AB and line DC are **parallel** and line AD and line BC are parallel.

6.10 CIRCLE AND OTHER GRAPHS

Circle graphs are used for **data** showing a relationship of the **parts to the whole**. Circle graphs are particularly useful for representing **percent**.

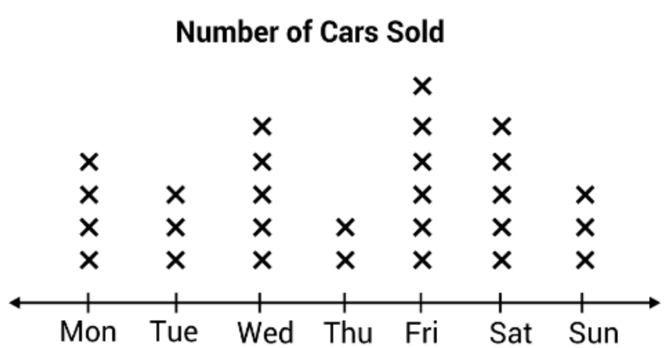


This circle graphs shows the favorite fruit chosen by students at the school. What percent chose Peaches? Answer: 25%

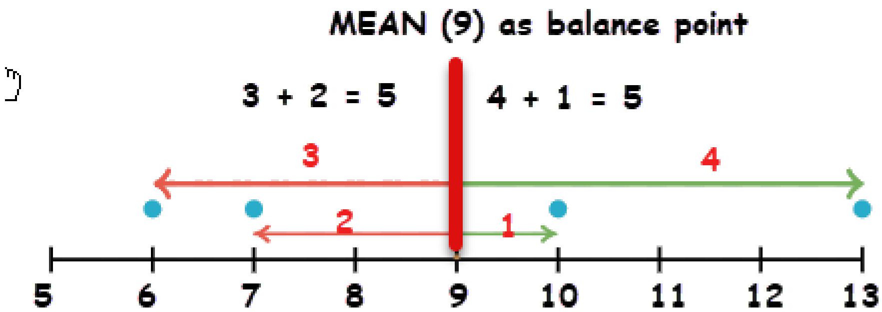
If there are 50 students in the school, how many chose oranges? Answer: 20

Line plots can be used to organize **numerical data**.

On which day were the fewest cars sold? Answer: Thursday.



6.11 MEAN AS BALANCE POINT



Find the mean of 4 numbers: 6, 7, 10, 13.

6 + 7 + 10 + 13 = **36**   36 ÷ 4 = **9**

To the left you can see the **mean 9** as **the balance point** – The distance of points **left** of the mean and points **right** of the mean both equal **5**.

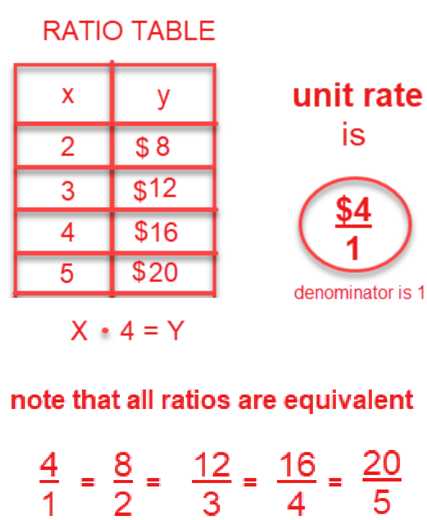
Mean, **median**, and **mode** are **measures of center** that are useful for describing the **average** for different situations.

Mean is useful when there are no values much higher or lower than the others.

Median (the middle number) is useful when there are a few numbers much higher or lower than the others that throw off the average. Example (2, 20, 21, 22, 24, 24, 28). What is the median? Answer: 22

Mode (think "**mo**" in **mode** for **most often**) is useful when there are **identical values** or you want to find the most popular. Example (2, 20, 21, 22, 24, 24, 28). What is the mode? Answer: 24

6.12 RATIOS ARE PROPORTIONAL RELATIONSHIPS



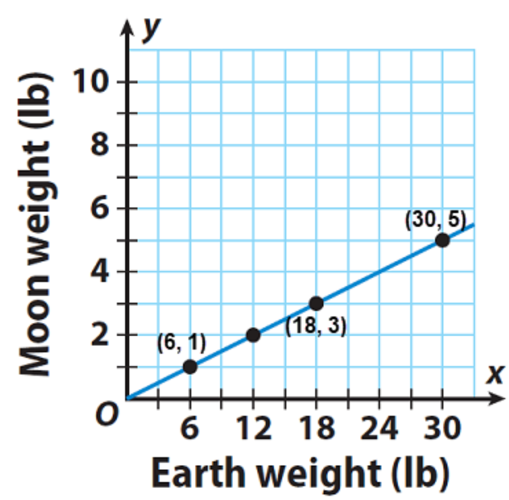
A **ratio** is a comparison of any two quantities.

Equivalent ratios – multiply each quantity by a **constant value.**

A **ratio table** includes equivalent ratios where one quantity is **multiplied** by a constant to get the second quantity.

UNIT RATES

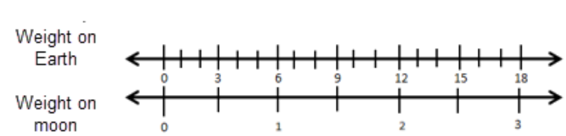
When dealing with ratios involving money, it's often helpful to find the **unit rate** (cost of one item). If given a ratio of $20/5, divide 20 by 5 to get the unit rate of $4/1. Unit rates have a **denominator of 1**.

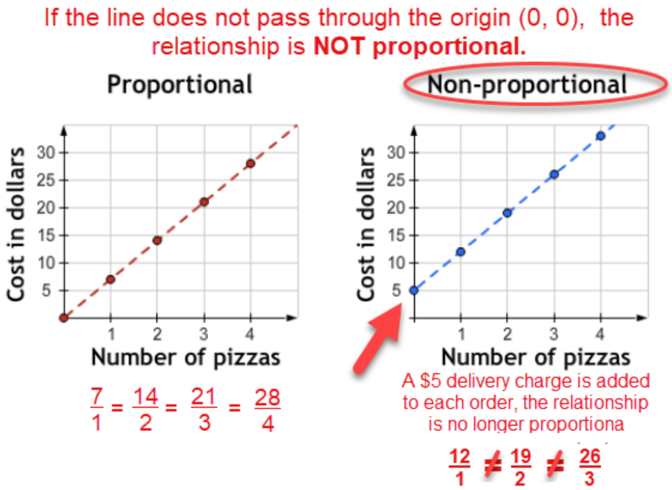


Earth weight x	6	12	18	24	30
Moon weight y	1	2	3	4	5

Proportional relationships can be described in words, tables, or graphed. →

An object's weight on Earth compared to weight on the moon can be represented by the ratio 6 : 1.





Proportional relationships are **graphed** with a **straight line** that if extended would pass through the **origin (0, 0).**

6.13 LINEAR EQUATIONS IN ONE VARIABLE

An **equation** contains and **equal sign**: *2x = 5* or *y – 8 = 2*

An **expression** represents a quantity and **does not** contain an equal sign: *5x* or *2 + x*

A **variable** is a symbol used when a **number is unknown**, as ***x*** is used in the expressions and equations above.

A **term** is a number, variable, product or quotient in an expression. The expression **3x + 5y – 2** contains **3 terms**, 3x, 5y, and 2

A **coefficient** is the **number in a term**. The term 3x includes the coefficient 3. 5 is the coefficient in the term 5y.

In addition to the properties listed in SOL 6.6, several more properties are listed in this standard. Students are **NOT meant to memorize** the names of these properties, but are meant to be **familiar** with these properties and able to use them to solve problems.

- **Commutative** property of **addition**: a + b = b + a
- **Commutative** property of **multiplication**: a · b = b · a

- **Associative** property of **addition**: (a + b) + c = a + (b + c)
- **Associative** property of **multiplication**: (ab)c = a(bc)

- **Distributive** property (over **addition/subtraction**): a(b+c) = ab + ac and a(b-c) = ab – ac

- **Identity** property of **addition**: a+0=a and 0+a=a
- **Identity** property of **multiplication** : a x 1 = a and 1 x a = a .

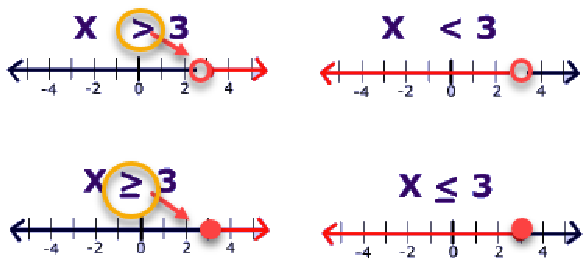
- **Inverse** property of **addition**: a + (-a) = 0
- **Inverse** property of **multiplication** (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \ and \ \frac{1}{a} \cdot a = 1.$

- **Multiplicative** property of **zero**: a · 0 = 0

- **Addition** property of **equality**: If $a = b$, then $a + c = b + c$.
- **Subtraction** property of **equality**: If $a = b$, then $a - c = b - c$.
- **Multiplication** property of **equality**: If $a = b$, then $a \cdot c = b \cdot c$.
- **Division** property of **equality**: If $a = b \ and \ c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.

- **Substitution** property: If $a = b$ then *b* can be substituted for *a* in any expression, equation or inequality.

6.14 LINEAR INEQUALITIES



Inequalities using the **< or > symbols** are represented on a number line with an open circle on the number and a shaded line over the solution set.

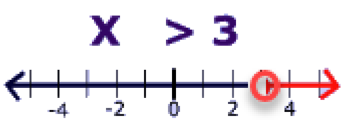
Inequalities using the **≤ or ≥ symbols** are represented on a number line with a **closed circle**

In addition to the properties listed in 6.13, students can use these properties:

- **Addition** property of **inequality**: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$ (this property also applies to $\leq$ $and\ \geq$).

- **Subtraction** property of **inequality**: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$ (this property also applies to $\leq and\ \geq$).

How would you show this inequality on a number line? 2 + x > 5 If you aren't sure, try plugging in some numbers for x. Does x=2 work? What about x = 4?



How would you show this inequality on a number line? *y* – 3 ≤ −6 Would y = -5 work? What about y = -3?

The inequality can be written $y \leq -3$. **Please note that the closed circle over the 3, meaning that 3 is included in the solution.**