MATH 6 – Summary of Math 6 Standards (2023)

Teachers: Please let me know if this guide is helpful, or how it could be improved.

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6.NS.1 Compare fractions, mixed numbers, decimals, and percent.

Percent (%) means "**per 100**". 10% = 10/100 = 1/10 = .1

?? What is 3/5 written as a percent and decimal? Answer: 3/5 = 6/10 = 60/100 = 60% = .60 = . 6

?? What is 73% written as a fraction and a decimal? Answer – 73/100 = .73

?? What about 162%? Answer - 162/100 = 1.62 (note that **100% = 1**, so more than 100% is more than 1)

When comparing fractions, it's often helpful to note if they are greater or less than ½ (example 3/7 < 4/8 < 5/9), or how close they are to 1 (6/7 < 7/8 < 8/9 < 9/10 because 1/7 > 1/8 > 1/9 > 1/10).

Proper fractions: numerator less than denominator, for instance ½

Improper fraction: numerator equal to or greater than denominator for example 3/2 or 3/3

Repeating decimals: 2/9 = 20/90 but as a decimal, the final 2 continues to repeat. This is written as .222**...** or **$0.\overline{2}$** to indicate a repeating 2. The **ellipse (…)** or **overline** means **repeating**.

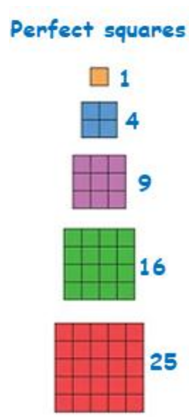
6.NS.2 REPRESENT, COMPARE, AND ORDER INTEGERS

Integers are **whole numbers** and can be positive or negative. {…-2, -1, 0, 1, 2, …}.

When comparing two **negative** integers, the one **closer to zero is greater** (-3 > -10 > -22).

Absolute value of an integer, written with **symbol ||** is its distance from zero on the number line. |-6| = 6, and |6| = 6. Absolute values are **always positive**. The absolute value of zero is zero.

6.NS.3 EXPONENTS AND PERFECT SQUARES



Any number (except 0) raised to **zero power is 1**. A positive number raised to the **one power** is the **number itself.**

5^3 - 5 is the **base,** 3 is the **exponent** = **5 x 5 x 5**

To indicate **multiplication,** a dot • can be used in place of **x**. 3^4 = 3 • 3 • 3 • 3 = 81

A **perfect square** is a number with a square root that is a whole number **–** These numbers are **perfect squares - 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324 , 361 (19 x 19), 400 (20 x 20). Notice the pattern – 1 + 3 = 4; 4 + 5 = 9; 9 + 7 = 16; 16 + 9 = 25; 25 + 11 = 36**

Powers of 10 (count the zeros) - 10^1 = 10; 10^2 = 100; 10^3 = 1000. What is 10^8? 100,000,000 (8 zeros)

6.CE.1 COMPUTATION – FRACTIONS and MIXED NUMBERS

FRACTIONS

Fraction in **simplest form –** divide numerator and denominator by **greatest common factor.** 10/15 = 2/3 (divide num. and den. by 5)

Multiply 2 fractions – multiply numerators to get numerator**, denominators** to get denominator**. ¼ x ½ = 1/8**

Multiply a fraction and a whole number - ½ x 8 = ½ x 8/1 = 8/2 = 4/1 = 4

Divide a fraction by another is the opposite, so **flip** the second fraction **–** ½ ÷ ¼ = ½ x 4/1 = 4/2 = 2/1 = 2

Check or **estimate** your answer – when you multiply 2 fractions, you end up with a **part of a part** – so the answer will be **less**. For example, ½ x ½ = ¼.

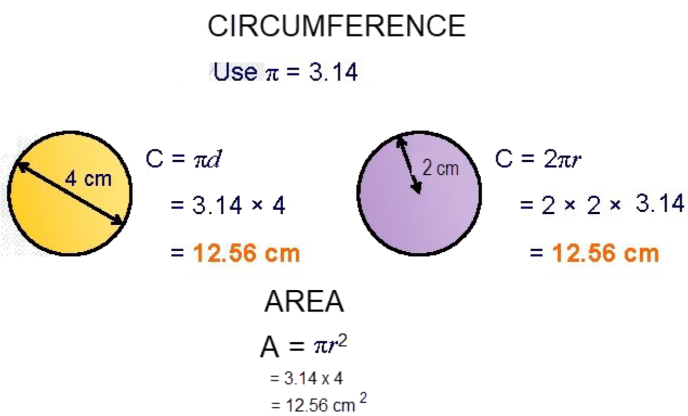
Multiply by a number between 0 and 1. 3 x 2/5 = 3/1 x 2/5 = 6/5 = $1\frac{1}{5}$ The result will be less than the number.

Divide by a number between 0 and 1. $3 \div \frac{2}{5} = 3 \times \frac{5}{2} = \frac{15}{2} = 7\frac{1}{2}$ The result will be greater than the number.

6.CE.2 COMPUTING WITH INTEGERS

Add, subtract, multiply, and divide two integers. Check to be sure these make sense.
Addition examples: 3 + -4 = -1; -3 + 5 = 2; -3 + -2 = -5
Subtraction examples: 2 – 5 = -3; What is 2 – (-5) = ? Answer 7 What is -2 – 7 =? Answer: 9 What is -2 – (-7) =? Answer 5
Note: -(-) is positive
Multiplication and **Division** - positive x/÷ negative = negative 6 · (-3) = -18; 18 ÷ (-3) = -6
negative x/÷ negative = positive -20 ÷ (-4) = 5; -6 · (-3) = 18

6.MG.1 CIRCUMFERENCE AND AREA OF CIRCLES



Chord: a straight line drawn between two points on a circle

Diameter: A chord that runs through the center of the circle

The value of **pi (π)** is the **ratio** of the **circumference** of a circle to its **diameter**.

pi (π) = approx. **3.14** or 22/7.

Circumference of a circle: **C = πd** (d is diameter) or **C = 2πr** (r is radius) . **Circumference** is approx. **three times** the **diameter**.

Area of a circle: **A = πr²** (r is radius)

When a circle has a radius of 2cm, note that area is 12.56cm^2 = circumference is 12.56cm. Why? Because 2^2 (used in area formula) = 2 x 2 (used in circumference formula). what is the area and circumference of a circle with a radius of 1cm?

Area = 3.14cm^2 Circumference = 6.28cm

?? What is the approx.. circumference and area of a circle with the radius of 3 cm ?

Answer - Approx **Circumference** = over 18 (6 x 3) More exactly: 6(diameter) x 3.14(pi) = 18.83 cm.

Area: approx. - r^2 = 9 so πr^2 is over 27cm^2 . More exactly, 9 x 3.14 = 28.26 cm^2

6.MG.2 AREA AND PERIMETER OF TRIANGLES AND PARALLELOGRAMS

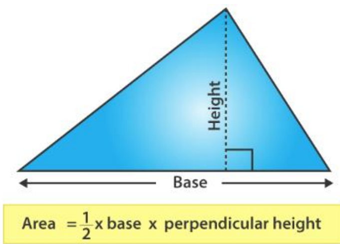
SQUARE – length of each **side** is **5cm**

- **Perimeter**: **4*s*** where *s* is side = 20cm
- **Area**: **S²** = 25cm^2

RECTANGLE – width = 3 cm, length = 4 cm

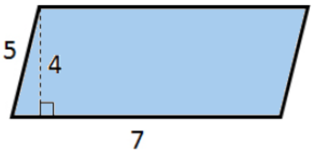
- Perimeter: **2*w* + 2*l*** = 14 cm
- Area: **length · width** = 12 cm^2

TRIANGLE



- **Perimeter**: add length of 3 sides
- **Area: A = ½ base x height**

Base = 6cm, height = 3cm   Area = 9cm^2



PARALLELOGRAM

- **Perimeter: 2*w* + 2*l*** = 10 + 14 = 24 cm
- **Area: base · height** = 7 x 4 = 28 cm^2

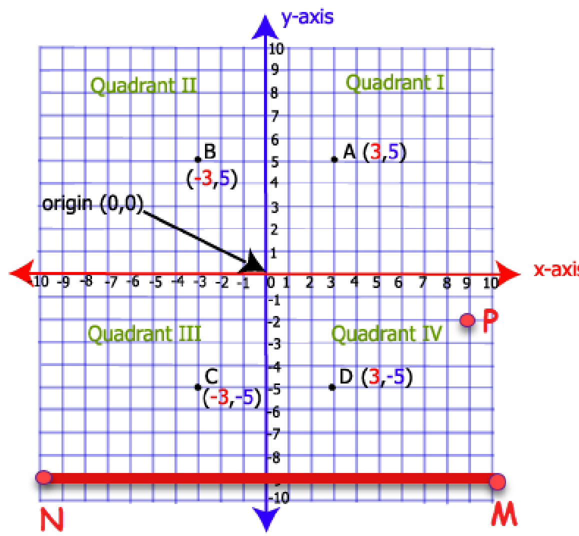
6.MG.3 COORDINATE PLANES

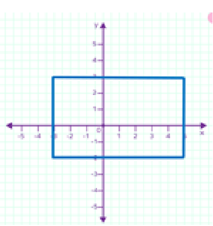
Know the origin (0, 0), x-axis, y-axis and quadrants I – IV (counterclockwise)

Points represented as **(x, y)**

What are the coordinates of point *P*?   answer: (9, -2)

What do all of the points an line NM share in common? For every point, y = -9.





Points with the same **x coordinate** form a **vertical line**.

What are the coordinates of the 4th vertex in a rectangle with vertices (-3, 3), (5, 3), (5, -2)?

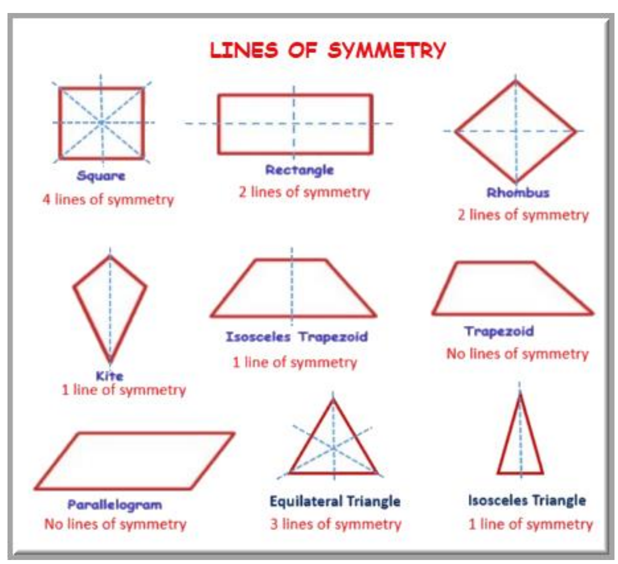
Answer (-3, -2)

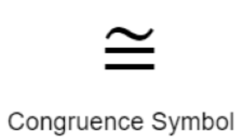
6.MG.4 CONGRUENCE OF SEGMENTS, ANGLES, AND POLYGONS

How many **lines of symmetry** in a square? Answer: 4
A rectangle or rhombus? Answer: 2
An equilateral triangle? Answer: 3
A parallelogram? Answer: 0

Regular Polygon - If all the sides and interior angles of the polygons are equal, they are known as **regular polygons**.

Which shapes in the image are **regular polygons**? Answer: square, rhombus, equilateral triangle.





The symbol for **congruency** is ≅

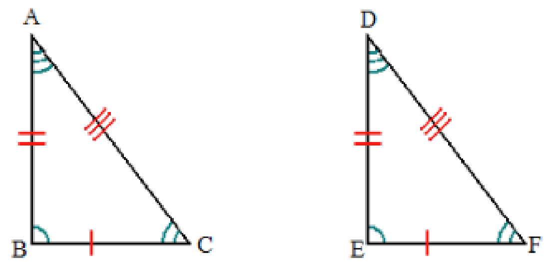
Congruent figures have **exactly the same size and shape**.

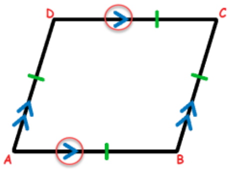
Congruent line segments have the same **length**.

Congruent angles have the same **measure** (in degrees)

Congruent polygons have the same number of sides, and the **corresponding sides and angles are congruent**.

These triangles are congruent. The hatch and angle marks indicate that line BC ≅ line EF, line BA ≅ line ED, angle D ≅ angle A etc.





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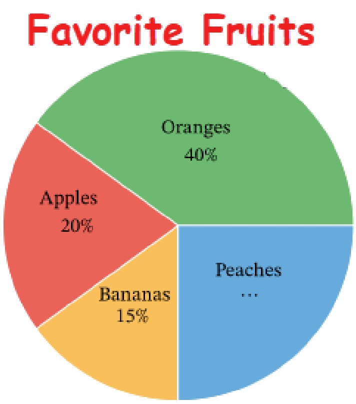
The **arrows** show that line AB and line DC are **parallel** and line AD and line BC are parallel.

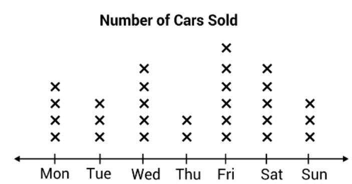
6.10 CIRCLE AND OTHER GRAPHS

Circle graphs are used for **data** showing a relationship of the **parts to the whole**. Circle graphs are particularly useful for representing **percent**.



This circle graphs shows the favorite fruit chosen by students at the school. What percent chose Peaches? Answer: 25%

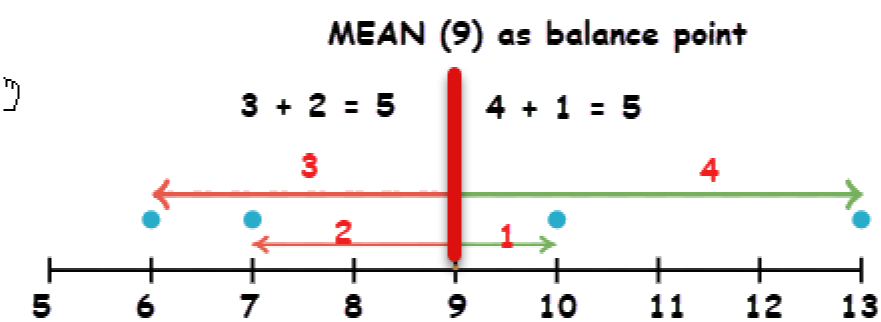
If there are 50 students in the school, how many chose oranges? Answer: 20



Line plots can be used to organize **numerical data**.

On which day were the fewest cars sold? Answer: Thursday.

6.PS.2 MEAN AS BALANCE POINT



Find the mean of 4 numbers: 6, 7, 10, 13.

6 + 7 + 10 + 13 = **36** 36 ÷ 4 = **9**

To the left you can see the **mean 9** as **the balance point** – The distance of points **left** of the mean and points **right** of the mean both equal **5**.

What happens to the mean if I add the number 14 to the set, so now (6, 7, 10, 13, 14)? Answer 10 (On a number line, the points would be 3 and 4 to the left of 10 and 3 and 4 to the right of 10, so 10 is the balance point) Of course, the easy way to figure out the mean is to add the numbers and divide by 5.

Important to understand the concept of the **mean as the balance point.**

What would the median be? Answer 10, the middle number.

What would the mode be? Answer – no mode

Mean, **median**, and **mode** are **measures of center** that are useful for describing the **average** for different situations.

Mean is useful when there are no values much higher or lower than the others.

Median (the middle number) is useful when there are a few numbers much higher or lower than the others that throw off the average. Example (2, 20, 21, 22, 24, 24, 28). What is the median? Answer: 22 (the middle number)

What about (2, 2, 20, 21, 22, 24, 24, 28)? Answer 21.5 (the average of the two middle numbers)

Mode (think "**mo**" in **mode** for **most often**) is useful when there are **identical values** or you want to find the most popular. Example (2, 20, 21, 22, 24, 24, 28). What is the mode? Answer: 24

What is the mode of (2, 20, 21, 22, 24, 28)? Answer: no mode

Outlier: a data value that is far from the other values, for example, in (7, 9, 13, 15, 64), the number 64 is the outlier.

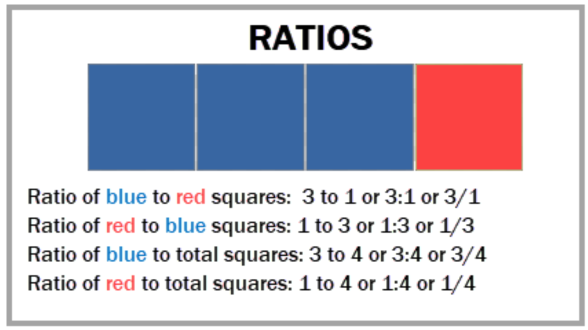
6.PFA.1 RATIOS ARE RELATIONSHIPS BETWEEN QUANTITIES

A **ratio** is a comparison between quantities
A ratio can be written as:

- a **fraction** – 2/3
- using a **colon** – 2:3
- using the word "**to**" – 2 to 3

In a classroom with 5 students with 2 girls and 3 boys

- The ratio of **girls** to **boys** is 2:3 or 2/3 or 2 to 3.
- The ratio of **boys** to **girls** is 3:2, or 3/2 or 3 to 2. Note that **ORDER MATTERS**.
- The ratio of **girls** to the **total** number of students is 2:5 or 2/5 or 2 to 5.
- The ratio of **boys** to **total** students is 3:5 or 3/5 or 3 to 5.



Note that in this example a ratio can be used to compare **2 parts** or a **part to a whole**.

?? How many **girls** in a classroom of **10 students** (same ratio of girls to boys - 2:3)? Answer – 4. Why? 2/3 = 4/6

?? How many **boys** in a classroom of **25 students**? Answer- 15. Why? 2/3 = 10/15

If the ratio of **boys** to the **total** number of students is **1:3,**

?? What is the ratio of **boys** to **girls**? Answer 1:2 (total = 3, 1 boy, 2 girls)

?? If there are 3 **boys**, how many **total** students are there? Answer – 9 (Why? 1/3 = 3/9)

?? If there are 5 **boys**, how many **girls** are there? Answer – 10 (Why? ½ = 5/10)

6.PFA.2 PROPORTIONAL RELATIONSHIPS

RATIO TABLE

x	y
2	$ 8
3	$12
4	$16
5	$20

X • 4 = Y

unit rate is $4 / 1 — denominator is 1

A **ratio** is a comparison of any two quantities.

Equivalent ratios – multiply each quantity by a **constant value. A rat**

io table includes equivalent ratios where one quantity is multiplied by a constant to get the second quantity.

UNIT RATES

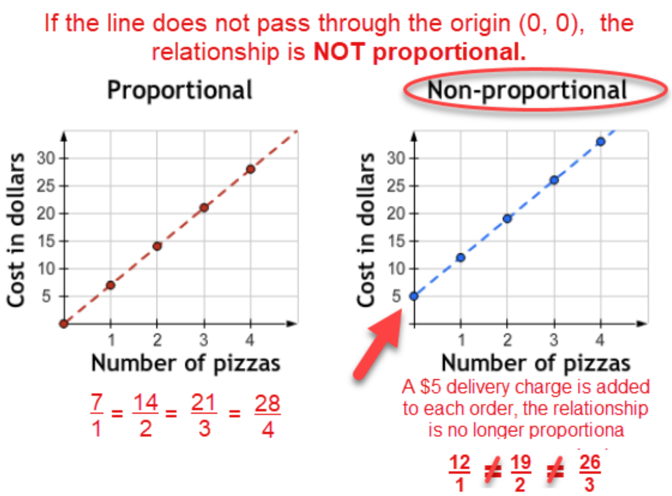
note that all ratios are equivalent

$$\frac{4}{1} = \frac{8}{2} = \frac{12}{3} = \frac{16}{4} = \frac{20}{5}$$

When dealing with ratios involving money, it's often helpful to find the **unit rate** (cost of one item). If given a ratio of $20/5, divide 20 by 5 to get the unit rate of $4/1. Unit rates have a **denominator of 1**.

Proportional relationships can be described in words, tables, or graphed. →

An object's weight on Earth compared to weight on the moon can be represented by the ratio 6 : 1.



Proportional relationships are **graphed** with a **straight line** that if extended would pass through the **origin (0, 0).**

6.PFA.3 LINEAR EQUATIONS IN ONE VARIABLE

An **equation** contains and **equal sign**: *2x = 5* or *y – 8 = 2*

An **expression** represents a quantity and **does not** contain an equal sign: *5x* or *2 + x*

A **variable** is a symbol used when a **number is unknown**, as ***x*** is used in the expressions and equations above.

A **term** is a number, variable, product or quotient in an expression. The expression **3x + 5y – 2** contains **3 terms**, 3x, 5y, and 2

A **coefficient** is the **number in a term**. The term 3x includes the coefficient 3. 5 is the coefficient in the term 5y.

In addition to the properties listed in SOL 6.6, several more properties are listed in this standard. Students are **NOT meant to memorize** the names of these properties, but are meant to be **familiar** with these properties and able to use them to solve problems.

- **Commutative** property of **addition**: a + b = b + a
- **Commutative** property of **multiplication**: a · b = b · a

- **Associative** property of **addition**: (a + b) + c = a + (b + c)
- **Associative** property of **multiplication**: (ab)c = a(bc)

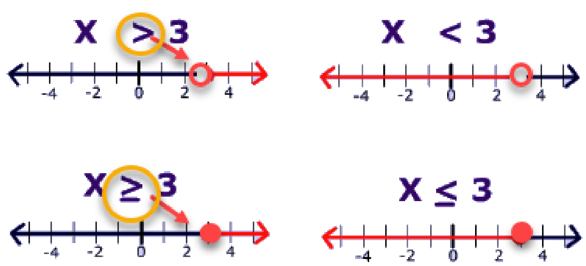
- **Distributive** property (over **addition/subtraction**): a(b+c) = ab + ac and a(b-c) = ab – ac

- **Identity** property of **addition**: a+0=a and 0+a=a
- **Identity** property of **multiplication** : a x 1 = a and 1 x a = a .

- **Inverse** property of **addition**: a + (-a) = 0
- **Inverse** property of **multiplication** (multiplicative inverse property): $a \cdot \frac{1}{a} = 1 \text{ and } \frac{1}{a} \cdot a = 1.$

- **Multiplicative** property of **zero**: a · 0 = 0
- **Addition** property of **equality**: If $a = b$, then $a + c = b + c$.
- **Subtraction** property of **equality**: If $a = b$, then $a - c = b - c$.
- **Multiplication** property of **equality**: If $a = b$, then $a \cdot c = b \cdot c$.
- **Division** property of **equality**: If $a = b \; and \; c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
- **Substitution** property: If $a = b$ then b can be substituted for *a* in any expression, equation or inequality.

6.PFA.4 LINEAR INEQUALITIES



Inequalities using the **< or > symbols** are represented on a number line with an open circle on the number and a shaded line over the solution set.

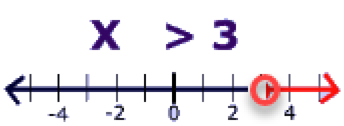
Inequalities using the **≤ or ≥ symbols** are represented on a number line with a **closed circle**

In addition to the properties listed in 6.13, students can use these properties:

- **Addition** property of **inequality**: If $a < b$, then $a + c < b + c$; if $a > b$, then $a + c > b + c$ (this property also applies to $\leq$ and $\geq$).

- **Subtraction** property of **inequality**: If $a < b$, then $a - c < b - c$; if $a > b$, then $a - c > b - c$ (this property also applies to $\leq$ and $\geq$).

How would you show this inequality on a number line? 2 + x > 5 If you aren't sure, try plugging in some numbers for x. Does x=2 work? What about x = 4?



How would you show this inequality on a number line? $y - 3 \leq -6$ Would y = -5 work? What about y = -3?

The inequality can be written $\boldsymbol{y \leq -3}$. **Please note that the closed circle over the 3, meaning that 3 is included in the solution.**